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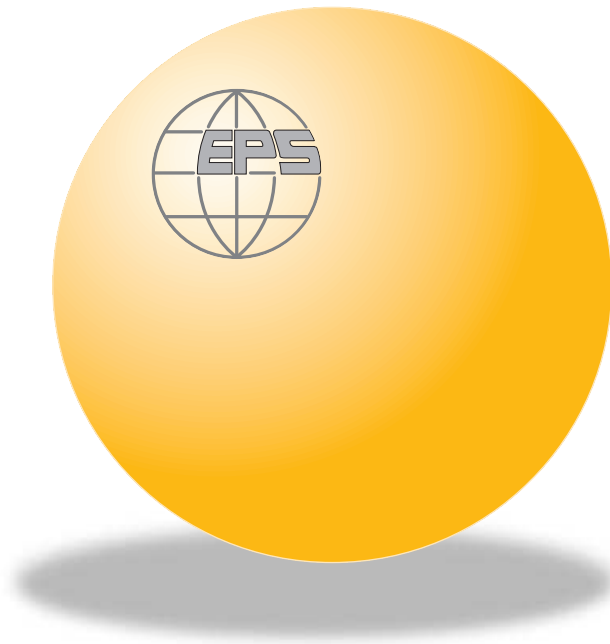
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Using synchronization to improve the forecasting of large relaxations in a cellular-automaton model

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Abstract. – A new forecasting strategy for stochastic systems is introduced. It is inspired by the concept of synchronization, developed in the area of Dynamical Systems, and by the earthquake forecasting algorithms in which different pattern recognition functions are used for identifying seismic premonitory phenomena. In the new strategy, copies (clones) of the original system (the master) are defined, and they are driven using rules that tend to synchronize them with the master dynamics. The observation of definite patterns in the state of the clones is the signal for connecting an alarm in the original system that efficiently marks the impending occurrence of a catastrophic event. The power of this method is quantitatively illustrated by forecasting the occurrence of the largest relaxations in the so-called Minimalist Model.

The last few decades have witnessed the irruption in geophysics literature of many new concepts coming from modern statistical physics, such as dynamical systems, chaos, fractals and self-organized criticality, among others [1]. This has been due to the genuine non-linearity of many geophysical phenomena, and in some cases, a specific geophysical phenomenon has been the origin of these nowadays widely known physical concepts. In this sense, think for instance of the dynamical meteorology equations in reference to chaos [2] or in the Gutenberg-Richter distribution in seismology in reference to self-organized criticality [3]. In the study of complex phenomena related to natural hazards (*e.g.*, earthquakes, forest fires and landslides), the so-called cellular-automaton models have resulted particularly useful [4,5].

We have recently introduced a model with episodic relaxations [6] as a sketch of an individual seismic fault. It is expressed by means of a one-parameter stochastic cellular automaton which, for its simplicity, is called the Minimalist Model (MM). The feasibility of forecasting the occurrence of the largest relaxations in the MM was initially assessed in a previous paper [7],

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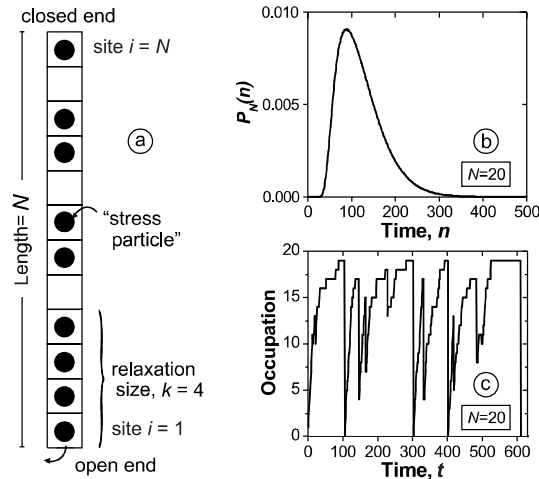(**) E-mail: `Amalio@unizar.es`

Fig. 1 – a) Sketch of the Minimalist Model (MM). b) Probability distribution function, $P_N(n)$, of the duration, n, of the cycles in a MM of size $N = 20$. c) Time evolution of the state of occupancy during four cycles in a MM of size $N = 20$.

where our results were certainly modest. In this letter, we return to this endeavour of forecasting synthetic series of large events, but now equipped with a new, more powerful, method. It is inspired, in part, by recent ideas on anticipated synchronization between chaotic oscillators, which constitutes an interesting topic in the area of Dynamical Systems [8–11]. There, the oscillator whose behavior one wishes to forecast is called *the master*, and the oscillator whose signal is equal to and precedes that of the master is called *the slave*. Its equivalent here will be named *the clone*. In spite of the obvious differences between a chaotic oscillator and the discrete MM, which is not governed by conventional differential equations, here we borrow several concepts from the former, which result in being notably useful for our purposes.

The MM [6] was devised in a spirit akin to the sandpile model of self-organized criticality [3]: It is a dissipative system loaded monotonically, and punctuated by episodic relaxations (earthquakes). This model is explained as follows (fig. 1a). Consider a one-dimensional vertical array of length N. Its sites will be labeled by an integer index i varying from 1 to N. This system performs two functions: it is loaded by receiving individual "stress particles" in the various sites of the array; and unloaded by emitting groups of particles through the first site, $i = 1$, which are called relaxations or earthquakes. These two functions proceed according to the following four rules:

1) The incoming particles arrive at the system at a constant time rate. Thus, the time interval between each two successive particles is the basic time unit in the evolution of the system.

2) All the sites in the array, from $i = 1$ to $i = N$, have the same probability of receiving a new particle. When a site receives a particle we say that it is occupied.

3) If a new particle comes to a site which is already occupied, this particle disappears from the system, so its stress is dissipated. Thus, a given site, i, can only be either empty, when no particle has come to it, or occupied when one or more particles have come to it.

4) When a particle goes to the site $i = 1$, a relaxation event occurs. Then, if all the successive sites from $i = 1$ up to $i = k$ are occupied, and the site $k + 1$ is empty, the effect of the relaxation is to unload all the sites from $i = 1$ up to $i = k$, so the size of this relaxation is k, $1 \leq k \leq N$. The remaining sites $i > k$ maintain their occupancy unaltered.
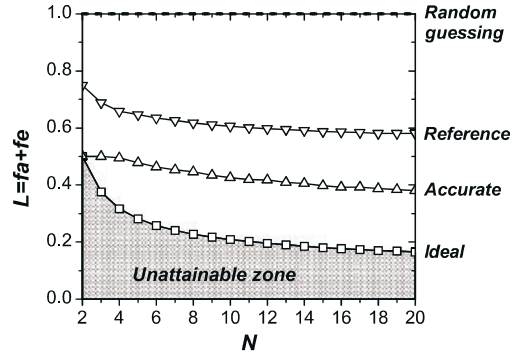
Fig. 2 – Loss function (L) obtained with the three forecasting methods used in this letter, for different system sizes ($2 \leq N \leq 20$). A random guessing strategy would render $L = 1$ for any N. The shadowed zone is unattainable for any forecasting strategy used in the MM, and the strategy that marks the lower limit of the reachable zone is called "Ideal". The "Reference" strategy was presented in [7], and the "Accurate" strategy is based on the synchronization of copies (clones) of the system with the real one (the master).

Thus, this model has only one parameter: N, the size of the array. The state of the system is given by stating which of the ($i > 1$) $N - 1$ sites are occupied. The prominent role given to the site $i = 1$ is analogous to that of a fault asperity, a particularly strong element in the system whose failure triggers the relaxation (earthquake) [12]. The relaxations of maximum size, $k = N$, are called characteristic, in analogy with the name of the largest earthquakes an individual fault may produce [13]. We will consider them as the target events to forecast in the MM.

Earthquake forecasting strategies frequently consist in connecting an alarm wisely before, and close to, the occurrence of the target events, usually the biggest earthquakes [14,15]. If an event occurs when the alarm is off, this is a failure to predict. Conversely, if the event occurs when the alarm is on, it is a successful prediction. Thus, the *fraction of error*, fe, is the ratio between the number of failures to predict and the total number of target events; and the *fraction of alarm*, fa, is the ratio between the time that the alarm was on and the total time of observation. In order to evaluate quantitatively the forecasting strategies that will be commented on later, we will use a loss function, L, in the form $L = fa + fe$. Other loss functions can be chosen (see [14], chapt. 5 by G. M. Molchan), but this will be used here for simplicity (as in [7]). Our purpose is obviously to obtain a value of L as low as possible.

A cycle of the MM lasts the time elapsed between two consecutive characteristic relaxations. Its duration is a stochastic variable, with a probability distribution function denoted by $P_N(n)$. This function has been proven useful as a renewal model to fit large-earthquake series on individual faults [16]. In fig. 1b, this distribution is plotted for $N = 20$. The knowledge of this distribution allows the setting of a first strategy in forecasting (introduced in [7]). It simply consists in connecting the alarm in each cycle at a constant value of n time steps after each characteristic relaxation, and identifying the value of n that renders the lowest value of L. In fig. 2, we have collected, for comparison, the results of the various strategies in terms of N. The curve labeled as "Reference" is the result of this strategy, and will be considered as a baseline to assess the actual merits of any other forecasting method [17].

Now, note that every cycle in the MM is composed of two independent and consecutive stages (fig. 1c). The first one, that will be called the stage of *loading*, starts just after the occurrence of a characteristic relaxation. During this stage, the total number of occupied sites

in the system grows, but not in a monotonic way, because the particles may be assigned to already occupied sites, and also because of the occurrence of small relaxations. When the load accumulated in the array reaches the maximum value of $N-1$ (when all the sites but the first one are occupied), this first stage ends and the second stage, that will be called the *hitting* stage, starts. In this second stage, the system resides in the state of maximum occupancy until a particle hits the first site. Then, a characteristic relaxation occurs, all the load in the system is lost, and a new cycle begins.

Both the time spent by the system in the loading stage, x, and in the hitting stage, y, are statistically distributed. The distribution of y, denoted by $P_2(y)$, is geometric. Its density is $P_2(y) = (1/N)(1 - 1/N)^{y-1}$, and its mean is $\langle y \rangle = N$. As the variables x and y are independent, $\langle n \rangle = \langle x \rangle + \langle y \rangle$. That is, the mean length of the cycles $\langle n \rangle$ is the sum of the mean lengths of the two stages. Now, we deduce the best result one could obtain in the forecasting of the characteristic events in this model. It is clear that the best L would be obtained *only if* we knew, in all the cycles, the instant at which the system concludes the stage of loading. But this would imply knowing the state of occupation of the system, which for most dynamic natural systems is not possible. For example, in real seismic faults, the state of stress at every point in the fault is not known. Here, however, and only for the purpose of establishing this ideal efficiency, we will assume that we know this piece of information. Thus, we can explore the result of L if we connect the alarm at a fixed value $y = y_0$ within the second stage of the cycles. One easily deduces that the minimum value of L is obtained for $y_0 = 0$, *i.e.* just after the end of the first stage, which indicates that this is a no-error forecasting strategy. The minimum value of L is $L_{\min} = N/\langle n \rangle$, and constitutes a rigorous lower bound for the accuracy of any forecasting strategy in the MM. The curve drawn in fig. 2 and labeled as "Ideal", represents this limit. For this reason, the area below that curve has been darkened to mark its inaccessibility.

Adopting the terminology used with chaotic oscillators, the minimalist system whose characteristic events we want to forecast will be called the *master* (M). It is important to realize that M, by definition, is not manipulable at all. It is also intrinsically opaque, in the sense that at any moment in the cycle we know neither the value of the load in the system nor the actual distribution of stress particles in the array. But, thanks to the occurrence of relaxations in M, it is possible to deduce some things about this distribution: just after the occurrence of a relaxation of size k, at least the $k+1$ first sites of M are empty. As a particular case, just after a characteristic relaxation occurs, the whole array is empty.

Now, we define copies of M which will be called C (for *clones*). In contrast to M, the C systems will be transparent and able to be manipulated; their only purpose is to help us to a better forecasting of M. Thus, at any moment, we will know how the load is distributed inside the clones and we will be able to modify it at will. During the course of a cycle, in principle, we will turn on the alarm when a certain pattern is observed in C, but it will be turned off if the occurrence of a small relaxation in M disproves the risk of an imminent characteristic relaxation. That would be a *false alarm*. Thus, during one cycle several alarm intervals may be defined which contribute to the fraction of alarm. M and C are driven subsequently, at the same rate, during the whole duration of a cycle. This means that for each stress particle randomly assigned to M, another one is randomly assigned to each C. All these assignments are independent of each other. Besides, the observed external effects caused on M by the addition of a particle (*i.e.* the possible occurrence of relaxations in M) are considered first. The new method of forecasting is explained in seven rules. Suppose, for simplicity, that only one clone is used.

1) At the beginning of a cycle, all the sites of C are empty (as in M).

2) If a particle assignment induces a relaxation of size $k < N$ in M, we impose that C evacuates
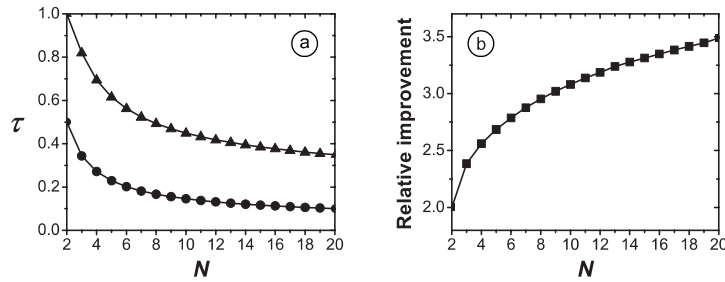
Fig. 3 – a) Fraction of time τ during which two MMs have the same number of occupied levels (dots: two independent masters; triangles: a master and a clone). b) Relative improvement of τ when using a master and a clone, given by the ratio of the master-clone τ to the master-master one.

its $k + 1$ lowest sites instead of receiving its corresponding particle assignment (in this way, both M and C will have their $k + 1$ lowest sites empty).

3) If a particle assignment induces no relaxation in M, we only allow the incoming particle in C to be dropped in any of the upper $N - 1$ sites, so that we preclude the occurrence of a relaxation in C.

4) When, after a particle assignment, M has no relaxation and C reaches its maximum state of occupancy, an *alarm* is turned on in M (because it is probable that M has also reached its maximum occupancy). The cycle continues, and we wait until the occurrence of the next relaxation in M. If this is a characteristic one, then this event is added to the general forecasting balance as a *success*, and its contribution to the alarm fraction is registered. And a new cycle begins. If it is not a characteristic one, pass to the fifth rule.

5) If, as in the fourth rule, the complete filling of C triggered an alarm in M but the next relaxation is of size $k < N$, this is a *false alarm*. So, the time elapsed since the alarm was turned on is registered as a contribution to the alarm fraction, the alarm is turned off, rule 2 is applied to C, and the same cycle continues.

6) If a characteristic relaxation occurs in M before C has reached its maximum occupancy, *i.e.* the alarm was off, then this outcome is taken as a *failure* to predict.

7) Finally, if, as will be described later, we are using an ensemble of clones, and one or more of them is completely full, the remaining incompletely filled clones continue to be charged, following the previous rules.

Note that these rules are designed in such a way that the dynamical evolution of each clone tends to be synchronized with that of the master. One possible measure of the degree of synchronization between master and clone is the fraction of time, τ, during which both systems have the same number of occupied levels. For two independent master systems, this synchronization takes place just by chance during a certain fraction of time (fig. 3a, lower curve). However, an M-C pair has a much greater τ for any N (fig. 3a, upper curve, and fig. 3b), being $\tau = 1$ (perfect synchronization) for $N = 2$.

In the field of Dynamical Systems the anticipated synchronization is attained by delayed feedback [8–11]. No delayed feedback is used here. This is because, in each time step, we first have to observe the external effects (relaxation or absence of relaxation) produced by M in order to apply the corresponding rule (2 or 3) to C. The complete filling of C is interpreted as an indication that M is probably full, and, consequently, that the occurrence of the characteristic relaxation is impending. However, statistically speaking, each C will complete the loading stage before M. This is because, in the second rule, the load lost by C is less than or equal to k, while M, for sure, loses k units. Therefore, the use of only one
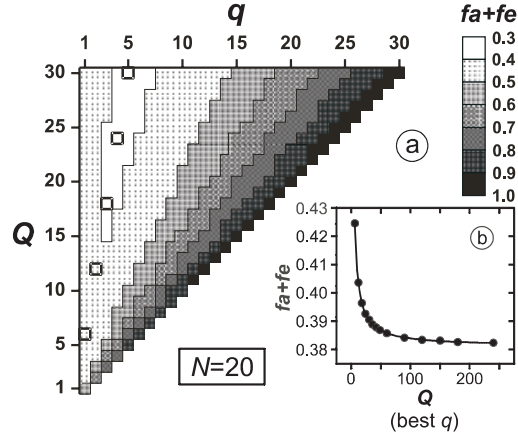
Fig. 4 – a) Loss function ($L = fa + fe$) resulting from different combinations of Q, the number of clones used, and q, the number of them awaited before connecting the alarm in the master array, for a system of size $N = 20$. The marked cells indicate the combinations that render the lowest L values per column q. b) Minimum loss function that may be obtained with a certain number Q of clones in a system of size $N = 20$. The first five points correspond to the marked cells of (a). The black dots result from Monte Carlo simulations, and the solid line is an exponential fit, which has the form $L = a \exp[b/(Q + c)]$, where a, b, and c are parameters. The extrapolation of this fit for $Q \to \infty$ renders an asymptotic value of $L = a = 0.381$, represented in the curve "Accurate" of fig. 2 for $N = 20$.

clone renders high values of the alarm fraction, and thus a high L. We have observed that in order to minimize L, it is more effective to employ an ensemble of several clones and wait until some of them have reached their maximum occupancy for turning the alarm on. When dealing with several (Q) clones, $1 \leq Q < \infty$, the seven above-mentioned rules for forecasting are applied to all the clones in an identical form. Each clone evolves independently of the others, and completes its total occupancy at a different time. We have numerically explored, for fixed N and Q, what is the number of fully loaded clones, q, one should wait for before connecting the alarm in M and obtaining the best prediction. This is illustrated in fig. 4a for $N = 20$. For a fixed N, the value of L improves as Q grows. However, the decrease in L reaches a saturation as shown in fig. 4b, where an exponential fit has been performed to identify the limit for $Q \to \infty$. These limit values for different system sizes have been drawn in fig. 2 forming the curve labeled as "Accurate". Note that the prediction equals the ideal one for $N = 2$, for which the degree of synchronization is perfect ($\tau = 1$), and that, for any N, L is always much lower than the reference one.

As far as we know, this is the first time that a forecasting method based on the synchronization of two purely stochastic systems (the master and the clone) has been developed. In conventional stochastic synchronization, the synchronization is only observed in the presence of a noise common to both systems [18, 19]. Earthquake forecasting algorithms frequently use several pattern recognition functions simultaneously, that, in real time, monitor the seismicity in order to search for seismic precursors. When several of them reach given thresholds, an alarm is turned on in the zone under study [14, 15]. This is the inspiration for using several clones: they, in some way, monitor the evolution of the master, and when a certain number of them reach their maximum occupancy, an alarm is turned on in the master. Our process of choosing the optimum q is equivalent to the learning phase of those algorithms [14]. However,

note that, in order to build copies of the system under study, one must be able to infer how the system works. This requirement, which is also present in the synchronization of chaotic oscillators [8–11], hinders the application of the strategy to complex dynamical systems in Nature.

To leave a quantitative flavour about the success of the clone-based strategy, we will briefly compare its results with our previous forecasting attempts on this model. In [7], we focused our attention on a system of size $N = 20$. Using the strategy considered as baseline, the minimum loss function is $L_{\min}(\text{reference}) = 0.578$. In [7] we also tried a biparametric strategy ($L_{\min}(\text{biparametric}) = 0.549$) and a triparametric one ($L_{\min}(\text{triparametric}) = 0.528$). So the maximum improvement in [7] was a modest 8.7%, when compared to the "reference" strategy. With the clone-based method, the minimum reachable value for $N = 20$ (when $Q \to \infty$) is $L_{\min}(\text{accurate}) = 0.381$ (fig. 4b), 34.1% better than the reference value, and halfway between the latter and the ideal one (fig. 2), $L_{\min}(\text{ideal}) = 0.165$. Moreover, in the new method the fraction of errors (fe) is systematically low: $fe = 0$ for $N = 2$, and increases slowly with N, with $fe = 0.094$ for $N = 20$ and $Q \to \infty$. In any case, it is important to bear in mind that the behaviour of a complex system cannot be predicted with absolute precision. One can attempt to reduce the errors in the forecasting, but not eliminate them completely [14].

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